4.1 Three Phase Induction Motor

The most common type of AC motor being used throughout the work today is the "Induction Motor". Applications of three-phase induction motors of size varying from half a kilowatt to thousands of kilowatts are numerous. They are found everywhere from a small workshop to a large manufacturing industry.

The advantages of three-phase AC induction motor are listed below:

- Simple design
- Rugged construction
- Reliable operation
- Low initial cost
- Easy operation and simple maintenance
- Simple control gear for starting and speed control
- High efficiency.

Induction motor is originated in the year 1891 with crude construction (The induction machine principle was invented by *NIKOLA TESLA* in 1888.). Then an improved construction with distributed stator windings and a cage rotor was built.

The slip ring rotor was developed after a decade or so. Since then a lot of improvement has taken place on the design of these two types of induction motors. Lot of research work has been carried out to improve its power factor and to achieve suitable methods of speed control.

4.2 Types and Construction of Three Phase Induction Motor

Three phase induction motors are constructed into two major types:

1. Squirrel cage Induction Motors
2. Slip ring Induction Motors

4.2.1 *Squirrel cage Induction Motors*

(a) Stator Construction

The induction motor stator resembles the stator of a revolving field, three phase alternator. The stator or the stationary part consists of three phase winding held in place in the slots of a laminated steel core which is enclosed and supported by a cast iron or a steel frame as shown in Fig: 4.1(a).

The phase windings are placed 120 electrical degrees apart and may be connected in either star or delta externally, for which six leads are brought out to a terminal box mounted on the frame of the motor. When the stator is energized from a three phase voltage it will produce a rotating magnetic field in the stator core.

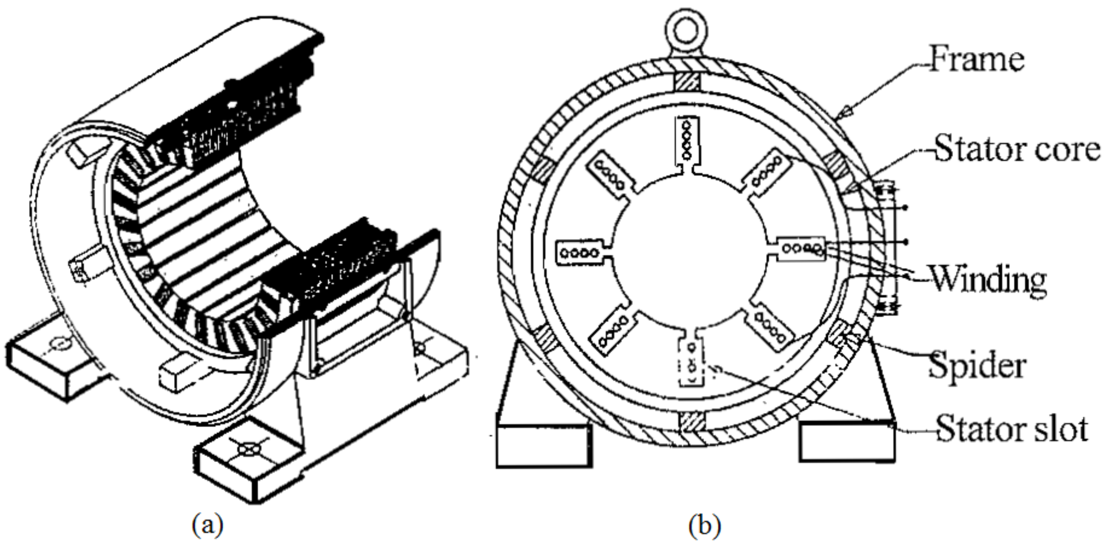


Fig: 4.1

(b) Rotor Construction

The rotor of the squirrel cage motor shown in Fig: 4.1(b) contains no windings. Instead it is a cylindrical core constructed of steel laminations with conductor bars mounted parallel to the shaft and embedded near the surface of the rotor core.

These conductor bars are short circuited by an end rings at both end of the rotor core. In large machines, these conductor bars and the end rings are made up of copper with the bars brazed or welded to the end rings shown in Fig: 3.1(b).In small machines the conductor bars and end rings are sometimes made of aluminium with the bars and rings cast in as part of the rotor core. Actually the entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

The rotor or rotating part is not connected electrically to the power supply but has voltage induced in it by transformer action from the stator. For this reason, the stator is sometimes called the primary and the rotor is referred to as the secondary of the motor since the motor operates on the principle of induction and as the construction of the rotor with the bars and end rings resembles a squirrel cage, the squirrel cage induction motor is used.

The rotor bars are not insulated from the rotor core because they are made of metals having less resistance than the core. The induced current will flow mainly in them. Also the rotor bars are usually not quite parallel to the rotor shaft but are mounted in a slightly skewed position. This feature tends to produce a more uniform rotor field and torque. Also it helps to reduce some of the internal magnetic noise when the motor is running.

(c) End Shields

The function of the two end shields is to support the rotor shaft. They are fitted with bearings and attached to the stator frame with the help of studs or bolts attention.

4.2.2 *Slip-ring Induction Motors*

(a) Stator Construction

The construction of the slip ring induction motor is exactly similar to the construction of squirrel cage induction motor. There is no difference between squirrel cage and slip ring motors.

(b) Rotor Construction

The rotor of the slip ring induction motor is also cylindrical or constructed of lamination.

Squirrel cage motors have a rotor with short circuited bars whereas slip ring motors have wound rotors having "three windings" each connected in star.

The winding is made of copper wire. The terminals of the rotor windings of the slip ring motors are brought out through slip rings which are in contact with stationary brushes as shown in Fig: 4.2.

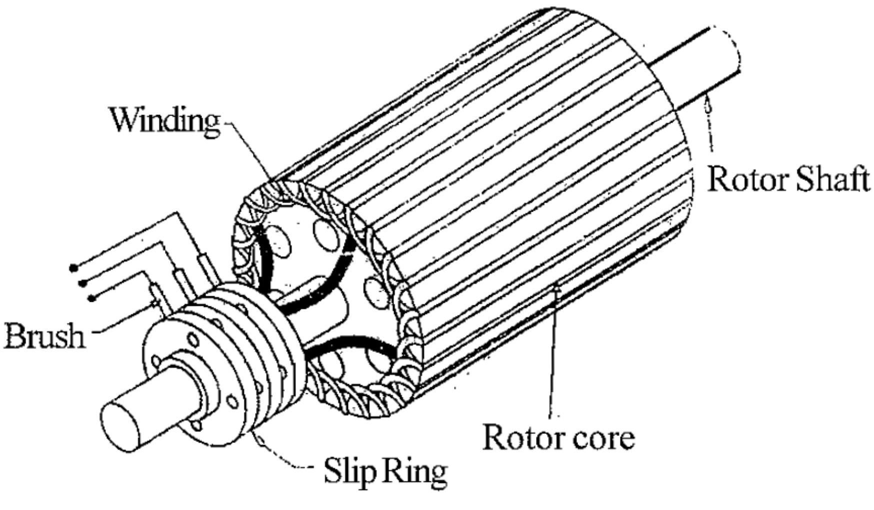


Fig: 4.2

THE ADVANTAGES OF THE SLIPRING MOTOR ARE

- It has susceptibility to speed control by regulating rotor resistance.
- High starting torque of 200 to 250% of full load value.
- Low starting current of the order of 250 to 350% of the full load current.

Hence slip ring motors are used where one or more of the above requirements are to be met.

4.2.3 Comparison of Squirrel Cage and Slip Ring Motor

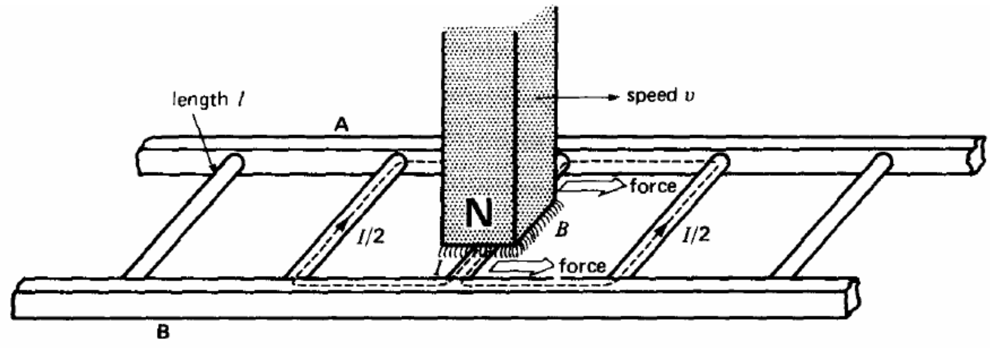
Sl.No.	Property	*Squirrel cage motor*	*Slip ring motor*
1.	**Rotor Construction**	*Bars are used in rotor. Squirrel cage motor is very simple, rugged and long lasting. No slip rings and brushes*	*Winding wire is to be used.* *Wound rotor required attention.* *Slip ring and brushes are needed* also need *frequent maintenance.*
2.	**Starting**	*Can be started by D.O.L., star-delta, auto transformer starters*	*Rotor resistance starter is required.*
3.	**Starting torque**	*Low*	*Very high*
4.	**Starting Current**	*High*	*Low*
5.	**Speed variation**	*Not easy, but could be varied in large steps by pole changing or through smaller incremental steps through thyristors or by frequency variation.*	*Easy to vary speed.* *Speed change is possible by inserting rotor resistance using thyristors or by using frequency variation injecting emf in the rotor circuit cascading.*
6.	**Maintenance**	*Almost ZERO maintenance*	*Requires frequent maintenance*
7.	**Cost**	*Low*	*High*

4.3 Principle of Operation

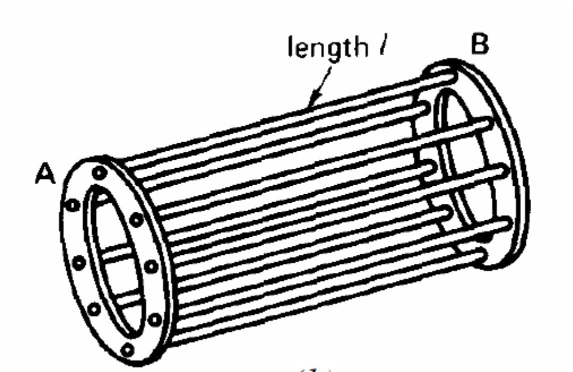
The operation of a 3-phase induction motor is based upon the application of Faraday Law and the Lorentz force on a conductor. The behaviour can readily be understood by means of the following example.

Consider a series of conductors of length l, whose extremities are short-circuited by two bars A and B (Fig.4.3 a). A permanent magnet placed above this conducting ladder, moves rapidly to the right at a speed v, so that its magnetic field B sweeps across the conductors. The following sequence of events then takes place:

1. A voltage E = Blv is induced in each conductor while it is being cut by the flux (Faraday law).
2. The induced voltage immediately produces a current I, which flows down the conductor underneath the pole face, through the end-bars, and back through the other conductors.
3. Because the current carrying conductor lies in the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force).
4. The force always acts in a direction to drag the conductor along with the magnetic field. If the conducting ladder is free to move, it will accelerate toward the right. However, as it picks up speed, the conductors will be cut less rapidly by the moving magnet, with the result that the induced voltage E and the current I will diminish. Consequently, the force acting on the conductors wilt also decreases. If the ladder were to move at the same speed as the magnetic field, the induced voltage E, the current I, and the force dragging the ladder along would all become zero.



(a)



(b)

Fig: 4.3

In an induction motor the ladder is closed upon itself to form a squirrel-cage (Fig.4.3b) and the moving magnet is replaced by a rotating field. The field is produced by the 3-phase currents that flow in the stator windings.

4.4 Rotating Magnetic Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns (Fig.4.4). Coils that are diametrically opposite are connected in series by means of three jumpers that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, which are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magneto motive forces that act in the same direction.

The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.

For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the centre of the magnetic poles. The result is illustrated in Fig.3.5. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.

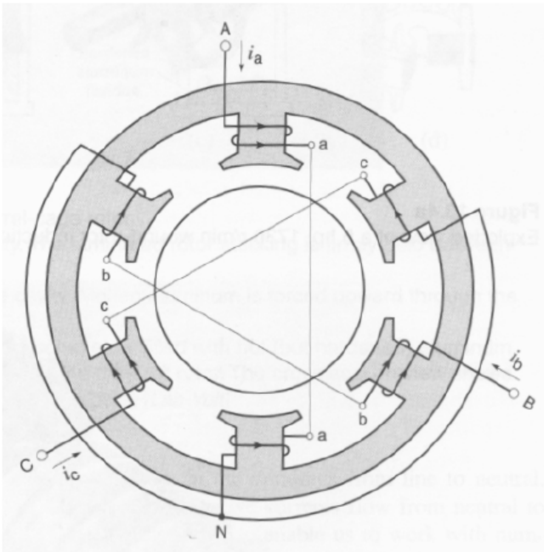


Fig: 4.4 Elementary stator having terminals A, B, C connected to a 3-phase source (not shown). Currents flowing from line to neutral are considered to be positive.

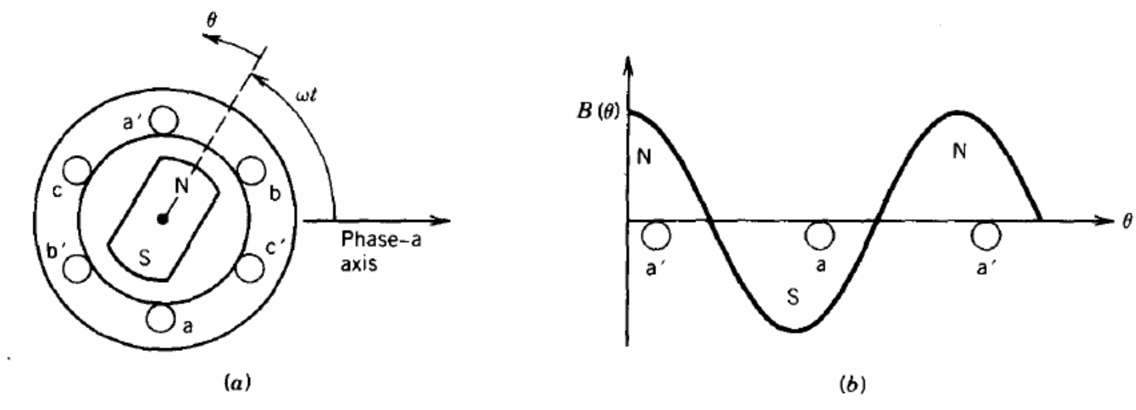


Fig: 4.5 Air gap flux density distribution.

The flux density distribution in the air gap can be expressed as:

$$B(\theta) = B_{\max} \cos \theta$$

The air gap flux per pole, ϕ_P, is:

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) lr d\theta = 2B_{\max} lr$$

Where,

l is the axial length of the stator.

r is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig.3.5). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa' will be maximum.

($= N\phi_P$ at $\omega t = 0^o$) (Fig.3.5a) and zero at $\omega t = 90^o$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt.

Hence,

$$\lambda_a(\omega t) = N\phi_p \cos \omega t$$

Therefore, the voltage induced in phase coil **aa'** is obtained from *Faraday law* as:

$$e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\max} \sin \omega t$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{\max} \sin(\omega t - 120)$$

$$e_c = E_{\max} \sin(\omega t + 120).$$

the *rms* value of the induced voltage is:

$$E_{rms} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44 fN\phi_p$$

Where f is the frequency in hertz. Above equation has the same form as that for the induced voltage in transformers. However, $Ø_P$ represents the flux per pole of the machine.

The above equation also shows the rms voltage per phase. The N is the total number of series turns per phase with the turns forming a concentrated full-pitch winding. In an actual AC machine each phase winding is distributed in a number of slots for better use of the iron and copper and to improve the waveform. For such a distributed winding, the EMF induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the EMF is less than their numerical sum when they are connected in series for the phase winding. A reduction factor K_W, called the winding factor, must therefore be applied. For most three-phase machine windings K_W is about 0.85 to 0.95.

Therefore, for a distributed phase winding, the rms voltage per phase is

$$Erms = 4.44 f N_{ph} \varphi_p K_W$$

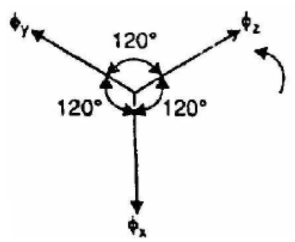
Where N_{ph} is the number of turns in series per phase.

4.5 Alternate Analysis for Rotating Magnetic Field

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 m where m is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in Fig. 4.6 (i). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as Ix, Iy and Iz [See Fig. 4.6 (ii)]. Referring to Fig. 4.6 (ii), the fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$
$$\phi_y = \phi_m \sin (\omega t - 120^\circ)$$
$$\phi_z = \phi_m \sin (\omega t - 240^\circ)$$



Here ϕ_m is the maximum flux due to any phase. Above figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 ϕ_m.

At instant 1 [See Fig. 4.6 (ii) and Fig. 4.6 (iii)], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5 ϕ_m as proved under:

At instant 1, $\omega t = 0°$. Therefore, the three fluxes are given by;

$$\phi_x = 0; \qquad \phi_y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

So,

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5\ \phi_m$$

At instant 2 [Fig: 4.7 (ii)], the current is maximum (negative) in ϕ_y phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5 ϕ_m as proved under:

At instant 2, $\omega t = 30°$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 30° = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90°) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210°) = \frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

Phasor sum of ϕ_x and ϕ_z, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that resultant flux is displaced 30° clockwise from position 1.

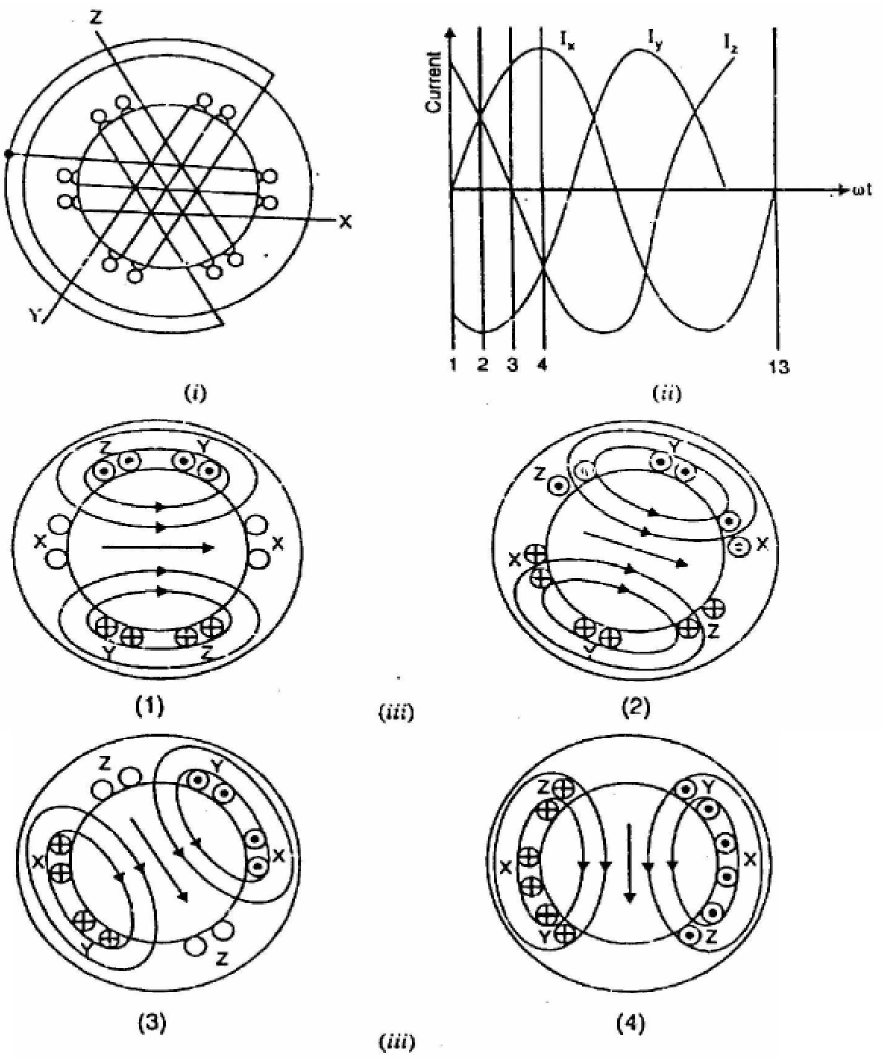


Fig: 4.6

At instant 3[Fig: 4.7 (iii)], current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc 0.866 × max. value). The magnitude of resultant flux is 1.5 ϕ_m as proved under:

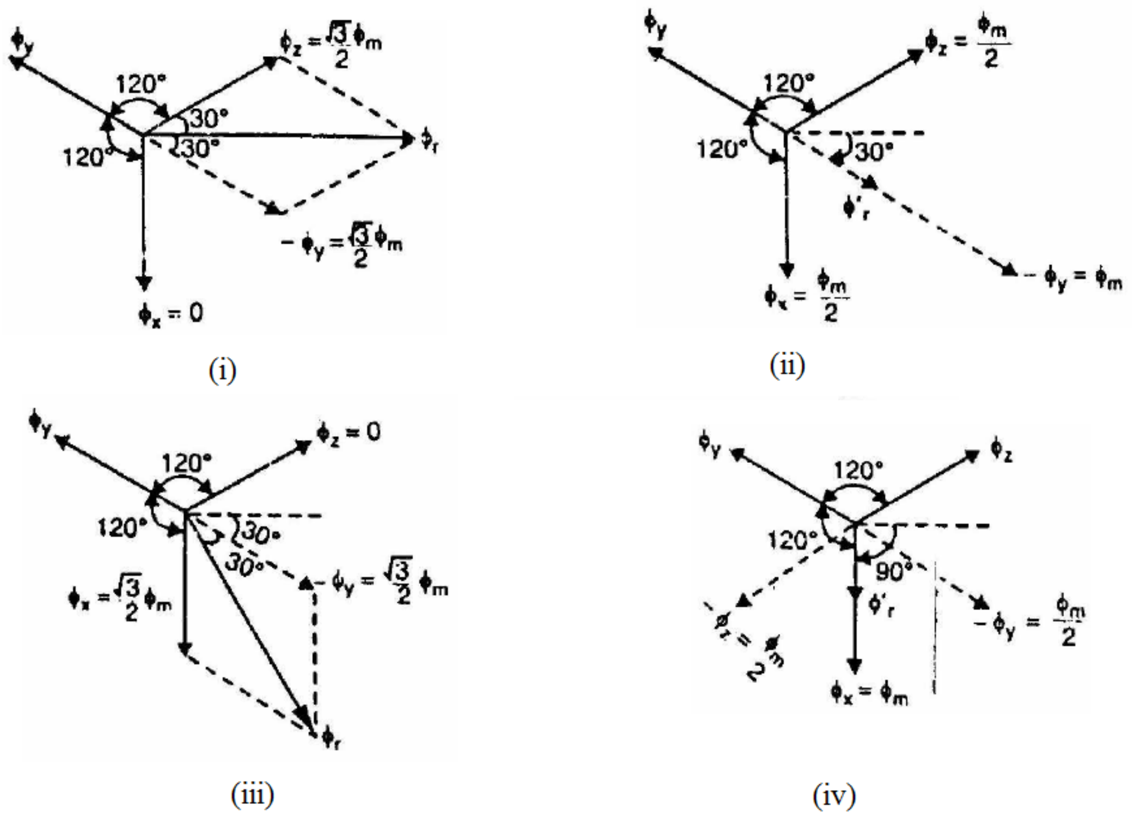


Fig: 4.7

At instant 3, $\omega t = 60°$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60° = \frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_y = \phi_m \sin(-60°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-180°) = 0$$

The resultant flux ϕ_r is the phasor sum of ϕ_x and $-\phi_y$ $(\because\ \phi_z = 0)$.

$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 1.5\ \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

At instant 4 [Fig: 4.7 (iv)], the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are 0.5 × max. value). This establishes a resultant flux downward as shown under:

At instant 4, ωt = 90°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90° = \phi_m$$

$$\phi_y = \phi_m \sin(-30°) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150°) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

Phasor sum of $-\phi_z$ and $-\phi_y$, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and ϕ_x, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5 ϕ_m, where ϕ_m is the maximum flux due to any phase).

4.5.1 Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns). Referring to Fig. 4.6 (ii), the time instant 4 represents the completion of one-quarter cycle of alternating current Ix from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 [Fig. 4.6 (ii)] or one complete cycle of current Ix from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \quad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

or $$\text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

or $$N_s = \frac{120\ f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

4.5.2 Direction of rotating magnetic field

The phase sequence of the three-phase voltage applied to the stator winding in Fig. 4.6 (ii) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counter clockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of

the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

4.5.3 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

$$\%\text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity $N_s - N$ is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

4.5.4 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;

$$\text{Frequency} = \frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding
P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is $N_s - N$. Consequently, the rotor current frequency f' is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{sN_sP}{120} \qquad \left(\because s = \frac{N_s - N}{N_s}\right)$$

$$= sf \qquad \left(\because f = \frac{N_sP}{120}\right)$$

i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency ($f' = sf = 1 \times f = f$).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

4.6 Equivalent Circuit of Three Phase Induction Motor

Fig. 4.10 (i) shows the equivalent circuit per phase of the rotor at slip s. The rotor phase current is given by;

$$I'_2 = \frac{s\,E_2}{\sqrt{R_2^2 + (s\,X_2)^2}}$$

Mathematically, this value is unaltered by writing it as:

$$I'_2 = \frac{E_2}{\sqrt{(R_2/s)^2 + (X_2)^2}}$$

As shown in Fig. 4.10 (ii), we now have a rotor circuit that has a fixed reactance X connected in series with a variable resistance R2/s and supplied with constant voltage E . Note that Fig. 4.10 (ii) transfers the variable to the resistance without altering power or power factor conditions.

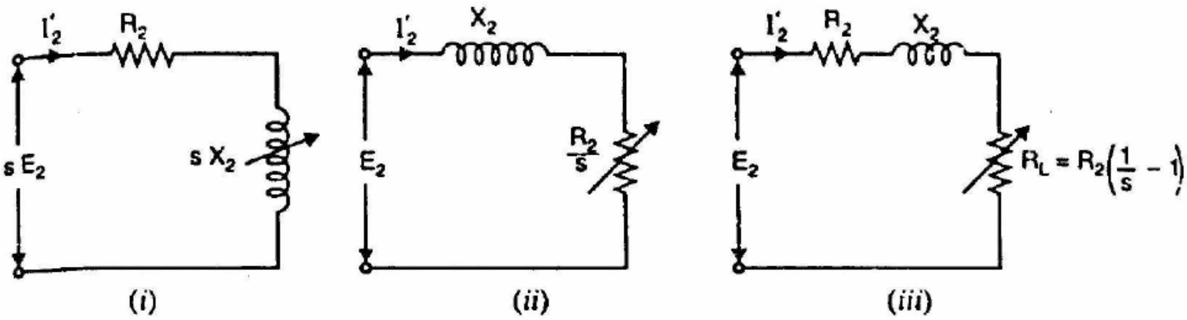


Fig: 4.10

The quantity R_2/s is greater than R_2 since s is a fraction. Therefore, R_2/s can be divided into a fixed part R_2 and a variable part $(R_2/s - R_2)$ i.e.,

$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$$

(i) The first part R_2 is the rotor resistance/phase, and represents the rotor Cu loss.

(ii) The second part $R_2\left(\frac{1}{s}-1\right)$ is a variable-resistance load. The power delivered to this load represents the total mechanical power developed in the rotor. Thus mechanical load on the induction motor can be replaced by a variable-resistance load of value $R_2\left(\frac{1}{s}-1\right)$. This is

$$\therefore \quad R_L = R_2\left(\frac{1}{s}-1\right)$$

Fig. 4.10 (iii) shows the equivalent rotor circuit along with load resistance R .

Now Fig: 4.11 shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance R_L given by;

$$R_L = R_2\left(\frac{1}{s}-1\right) \qquad \text{----- (i)}$$

The circuit shown in Fig. 4.11 is similar to the equivalent circuit of a transformer with secondary load equal to R_2 given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio K (= E_2/E_1).

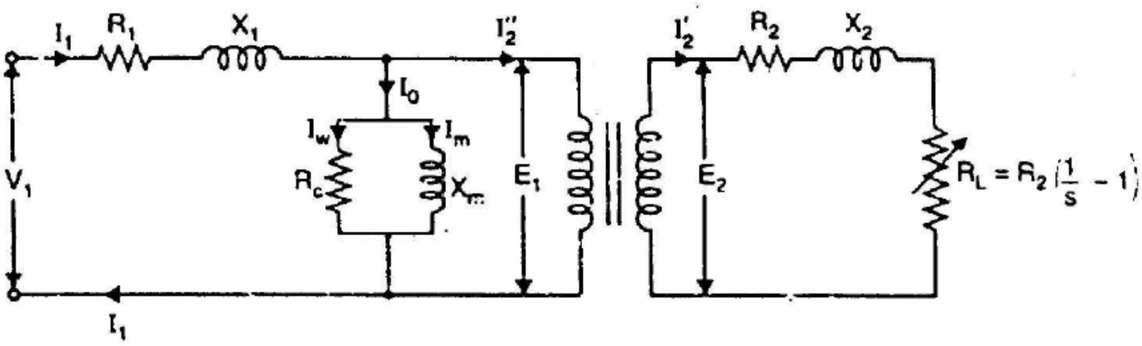


Fig: 4.11

Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load R_L given by eq. (i). The power delivered to R_L represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. 4.11 is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio K. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by K2 whereas current should be multiplied by K. The equivalent circuit of an induction motor referred to primary is shown in Fig. 4.12.

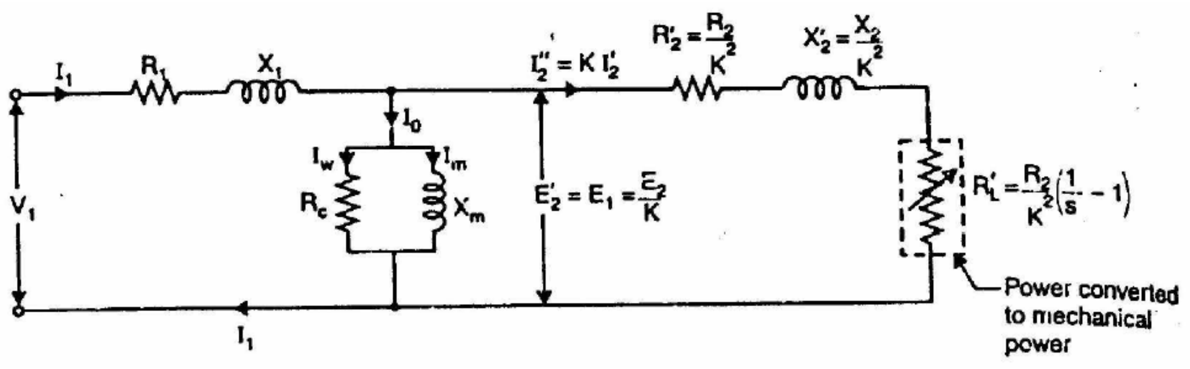


Fig: 4.12

Note that the element (i.e., R'_L) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

(i) At no-load, the slip is practically zero and the load R'_L is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.

(ii) At standstill, the slip is unity and the load R'_L is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.

(iii) When the motor is running under load, the value of R'_L will depend upon the value of the slip s. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.

(iv) The equivalent electrical resistance R'_L related to mechanical load is slip or speed dependent. If the slip s increases, the load R'_L decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor shaft.

4.7 Power and Torque Relations of Three Phase Induction Motor

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. 4.13 shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

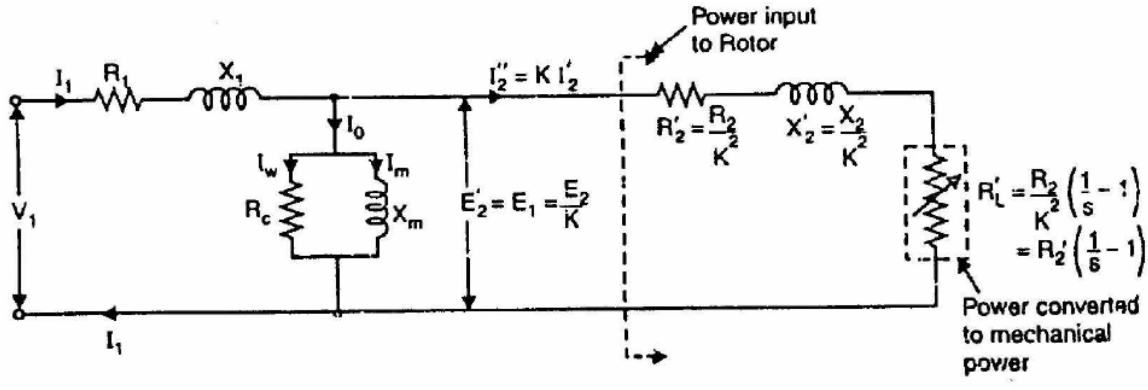


Fig: 4.13

(i) Total electrical load $= R'_2\left(\frac{1}{s}-1\right)+R'_2=\frac{R'_2}{s}$

Power input to stator $= 3V_1 I_1 \cos\phi_1$

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input $= \frac{3(I''_2)^2 R'_2}{s}$

Rotor Cu loss $= 3(I''_2)^2 R'_2$

Total mechanical power developed by the rotor is

P_m = Rotor input – Rotor Cu loss

$$= \frac{3(I''_2)^2 R'_2}{s} 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2\left(\frac{1}{s}-1\right)$$

This is quite apparent from the equivalent circuit shown in Fig: 4.13.

(iii) If T_g is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1\right) = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1-s}{s}\right) = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1-s}{s}\right) = \frac{2\pi N_s (1-s)\, T_g}{60} \qquad [\because N = N_s(1-s)]$$

$$\therefore \quad T_g = \frac{3(I''_2)^2 R'_2/s}{2\pi N_s/60} \quad \text{N - m}$$

or $$T_g = 9.55 \frac{3(I''_2)^2 R'_2/s}{N_s} \quad \text{N - m}$$

Note that shaft torque T_{sh} will be less than T_g by the torque required to meet windage and frictional losses.

4.8 Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \quad \text{watts}$$

where N = speed of the motor in r.p.m.
T = torque developed in N-m

$$\therefore \quad T = \frac{60}{2\pi}\frac{P}{N} = 9.55\frac{P}{N} \quad \text{N - m}$$

If the gross output of the rotor of an induction motor is P_m and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55 \frac{P_m}{N} \quad \text{N - m}$$

Similarly, $$T_{sh} = 9.55 \frac{P_{out}}{N} \quad \text{N - m}$$

Note. Since windage and friction loss is small, $T_g = T_{sh}$,. This assumption hardly leads to any significant error.

4.9 Rotor Output

If T_g newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi N T_g}{60} \quad \text{watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed Ns.

$$\therefore \quad \text{Rotor input} = \frac{2\pi N_s T_g}{60} \quad \text{watts}$$

$$\therefore \quad \text{Rotor Cu loss} = \text{Rotor input} - \text{Rotor output}$$

$$= \frac{2\pi T_g}{60}(N_s - N)$$

(i) $$\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \quad \text{Rotor Cu loss} = s \times \text{Rotor input}$$

(ii) Gross rotor output, P_m = Rotor input – Rotor Cu loss

= Rotor input – s × Rotor input

$$\therefore \quad P_m = \text{Rotor input } (1 - s)$$

(iii) $$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

(iv) $$\frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1 - s}$$

It is clear that if the input power to rotor is “Pr” then “s.Pr” is lost as rotor Cu loss and the remaining (1 - s) Pr is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses arc neglected, then,

Gross rotor output = Useful output

Rotor input = Stator input

$$\therefore \quad \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is 1 – s. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is = 1 – 0.125 = 0.875 or 87.5%.

4.10.1 Torque Equations

The gross torque Tg developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor input}}{2\pi\, N_s} \qquad \ldots N_s \text{ is r.p.s.}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi\, N_s} \qquad \ldots N_s \text{ is r.p.s.}$$

Now $\quad \text{Rotor input} = \dfrac{\text{Rotor Cu loss}}{s} = \dfrac{3(I'_2)^2 R_2}{s} \qquad$ (i)

As shown in Sec. 8.16, under running conditions,

$$I'_2 = \frac{s\, E_2}{\sqrt{R_2^2 + (s\, X_2)^2}} = \frac{s\, K\, E_1}{\sqrt{R_2^2 + (s\, X_2)^2}}$$

where $\quad K = \text{Transformation ratio} = \dfrac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \quad \text{Rotor input} = 3 \times \frac{s^2\, E_2^2\, R_2}{R_2^2 + (s\, X_2)^2} \times \frac{1}{s} = \frac{3\, s\, E_2^2\, R_2}{R_2^2 + (s\, X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

Also $\quad \text{Rotor input} = 3 \times \dfrac{s^2\, K^2\, E_1^2\, R_2}{R_2^2 + (s\, X_2)^2} \times \dfrac{1}{s} = \dfrac{3\, s\, K^2\, E_1^2\, R_2}{R_2^2 + (s\, X_2)^2}$

(Putting me value of I'_2 in eq.(i))

$$\therefore \quad T_g = \frac{\text{Rotor input}}{2\pi\, N_s} = \frac{3}{2\pi\, N_s} \times \frac{s\, E_2^2\, R_2}{R_2^2 + (s\, X_2)^2} \qquad \ldots\text{in terms of } E_2$$

$$= \frac{3}{2\pi\, N_s} \times \frac{s\, K^2\, E_1^2\, R_2}{R_2^2 + (s\, X_2)^2} \qquad \ldots\text{in terms of } E_1$$

Note that in the above expressions of T_g, the values E_1, E_2, R_2 and X_2 represent the phase values.

4.10.2 Rotor Torque

The torque T developed by the rotor is directly proportional to:

(i) rotor current

(ii) rotor e.m.f.

(iii) power factor of the rotor circuit

$$\therefore \quad T \propto E_2 I_2 \cos\phi_2$$

or $$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill

E_2 = rotor e.m.f. at standstill

$\cos\phi_2$ = rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

4.10.3 Starting Torque (Ts)

Let,

E_2 = rotor e.m.f. per phase at standstill

X_2 = rotor reactance per phase at standstill

R_2 = rotor resistance per phase

Rotor impedance/phase, $Z_2 = \sqrt{R_2^2 + X_2^2}$ ...at standstill

Rotor current/phase, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

Rotor p.f., $\cos\phi_2 = \dfrac{R_2}{Z_2} = \dfrac{R_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

$\therefore$ Starting torque, $T_s = K E_2 I_2 \cos\phi_2$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Generally, the stator supply voltage V is constant so that flux per pole ϕ set up by the stator is also fixed. This in turn means that e.m.f. E_2 induced in the rotor will be constant.

$$\therefore \quad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R_2 and X_2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that $K = 3/2 \pi N_s$.

$$\therefore \quad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here N_s is in r.p.s.

4.10.4 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now $$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \qquad \text{(i)}$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2} \right] = 0$$

or $$R_2^2 + X_2^2 = 2R_2^2$$

or $$R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque, $\phi_2 = 45°$ and rotor power factor is 0.707 lagging.

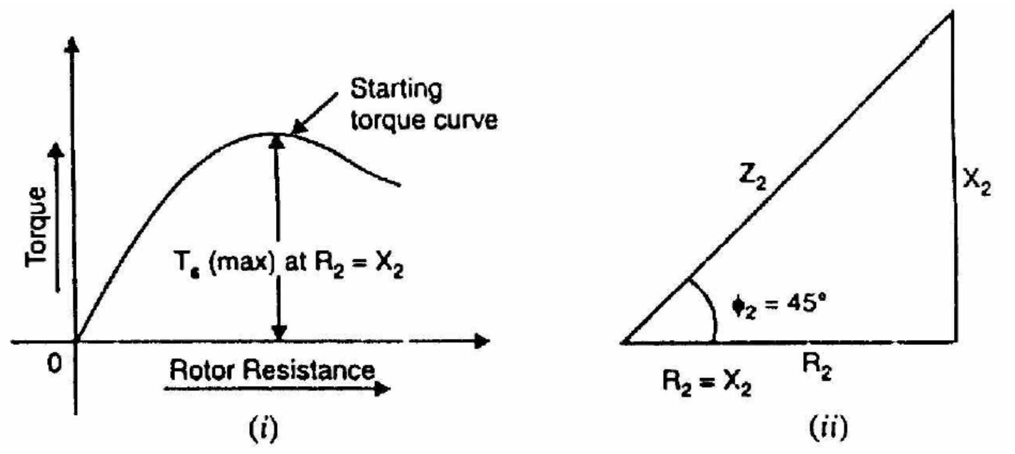


Fig: 4.14

Fig. 4.14 shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when $R_2 = X_2$. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.